1. Maria is preparing for a graduation party. She plans to invite 60 people and has budgeted $275 for the food. The caterer sells pans of pasta that serves 10 people for $55 each and trays of sandwiches that serves 6 people for $22 each.
   a. Define your variables and write a system of two linear equations in two variables to represent this problem situation.

   b. Graph the system of equations.

   c. Estimate the solution to the system of equations.
d. Solve the system using the linear combination method.

e. Solve the system using the substitution method.

f. Interpret the results.
2. Happy’s Hamburgers is analyzing their revenue and their expenses. The chief financial officer has determined that their revenue is modeled by \( y = 2.44x - \frac{x^2}{20000} - 5000 \) and their expenses are modeled by \( y = 5000 + 0.56x \), where \( x \) represents the number of hamburgers sold.

   a. Use substitution to write a new equation that can be used to solve the system.

   b. Use the quadratic formula to solve the resulting equation for \( x \). Round your answer to the nearest whole number and explain why it makes sense to do so.

   c. Calculate the corresponding value(s) for \( y \). Determine the solution(s) to the system of equations.
d. Graph each equation of the system and calculate the point(s) of intersection. Use the following values for your window: \( x: [0, 50,000] \) and \( y: [0, 30,000] \)

```
0 10000 20000 30000 40000 50000
```

```
40000 35000 30000 25000 20000 15000 10000 5000 0
```

```
10000
```

```
20000
```

```
30000
```

```
40000
```

```
45000
```

```
60000
```

e. What do you notice about the solutions that you calculated algebraically and graphically?

f. Interpret the results.
We Can Work It Out
Solving Systems of Three Equations Algebraically

1. Your parents have received an inheritance of $30,000. They want to put some of the money into a savings account that earns 1.5% interest annually and invest the rest in certificates of deposit (CDs) and bonds. A broker tells them that CDs pay 4% interest annually and bonds pay 5% interest annually. They want to earn $800 interest per year, and they want to invest twice as much money in CDs as in bonds. How much should they put in each type of investment?

   a. Define your variables and formulate a system of three linear equations in three variables to represent this situation.

   b. The equation \( c = 2b \) is already solved for a variable. Substitute 2b for c in the other two equations to create a system of two equations in two variables.
c. Solve the resulting system. Explain what your solution represents in terms of the problem situation.

2. You are helping your Grandmother with her flower garden. She decides to add some clematis vines, rose bushes, and peony plants to her garden. The price of a clematis vine is $3 more than twice a peony plant. The price of a rose bush is $5 more than a clematis vine. She decides to buy 3 clematis plants, 4 rose bushes and 2 peony plants for a total of $233.

a. Write a system of three linear equations in three variables to represent this situation. Define your variables.
b. Calculate the price of each item. Use substitution to solve the system of three linear equations in three variables.

3. John has 15 bills in his wallet that total $145. He has a mix of five-dollar bills, ten-dollar bills, and twenty-dollar bills. The number of ten-dollar bills is one less than twice the number of twenty-dollar bills.
   a. Write a system of three linear equations in three variables to represent this situation. Be sure to define your variables.
b. How many of each denomination does John have in his wallet? Solve this system using Gaussian elimination.
Step Inside the Matrix
Introduction to Matrices and Matrix Operations

1. Mr. Ramirez owns a video store and uses a matrix to keep track of the videos sold. The genres are Comedy (C), Science Fiction (SF), Action (A), Romance (R), and Television Series (TV). They are available in two formats, DVD or Blue Ray.

   \[ A = \begin{bmatrix} \text{C} & \text{SF} & \text{A} & \text{R} & \text{TV} \\ \text{DVD} & 42 & 38 & 51 & 30 & 22 \\ \text{Blue Ray} & 71 & 49 & 82 & 56 & 18 \end{bmatrix} \]

   a. Describe what each row and column represents in terms of the problem situation.

   b. What are the dimensions of this matrix?

   c. Determine the number that is in each location. Describe the meaning of the matrix element in terms of the problem situation.

   1. The element \( a_{24} \)

   2. The element \( a_{12} \)

   3. The element \( a_{11} \)

   d. What matrix element represents the number of Action movies on Blue Ray? Write your answer in matrix notation.
e. Mr. Ramirez also keeps track of revenue. He charges $15 for each DVD and $20 for each Blue Ray. Determine the revenue generated by each genre.

f. Determine the revenue generated by each video format.

g. Determine the total revenue generated by video sales.
2. Jenny buys her three favorite cereals and reads the labels to analyze their nutrition information. Happy O’s contain 6 grams of fiber and 4 grams of protein per serving. Blueberry Bites contain 5 grams of fiber and 3 grams of protein per serving. Cinnamon Squares contain 7 grams of fiber and 2 grams of protein per serving.

a. Organize this information into a “Nutritional Information” matrix.

b. Over the course of a week, Jenny eats three servings of Happy O’s, two servings of Blueberry Bites, and two servings of Cinnamon Squares. Organize this information into a “Servings” matrix. First, determine the dimensions the matrix should be so that you can multiply it by the “Nutritional Information” matrix.

c. Determine the dimensions of the product of the two matrices.

d. Calculate the product of the matrices to determine the amount of weekly fiber and protein Jenny is getting from the cereal she eats.
Another Tool in the Toolbox
Solving Matrix Equations

1. Consider the system of three linear equations in three variables

\[
\begin{align*}
2x + 2z &= 2 \\
5x + 3y &= 4 \\
3y - 4z &= 4
\end{align*}
\]

a. Write a matrix equation for the system in the form \(A \cdot X = B\).

b. Use technology to determine \(A^{-1}\).

c. Solve the matrix equation.
2. Maria is helping with a school fundraiser by selling fruit baskets. Basket A contains 3 apples, 2 pears, and 4 oranges and sells for $9.65. Basket B contains 4 apples, 3 pears, and 3 oranges and sells for $10.70. Basket C contains 2 apples, 2 pears, and 2 oranges and sells for $6.30. What is the cost of each apple, pear and orange?

a. Write a system of equations to represent the scenario. Define your variables.

b. Write the system of equations as a matrix equation.

c. Calculate the solution to the system of linear equations by using technology with matrices.
3. A middle school theater department sells tickets for their upcoming production. A child’s ticket costs $3.50, a student ticket costs $5, and an adult ticket costs $8.50. They sell the same number of student tickets as adult tickets. They sold a total of 82 tickets and total income from ticket sales is $495.

   a. Write a system of three linear equations in three variables to represent this scenario. Define your variables.

   b. Write the system of equations as a matrix equation.

   c. How many of each ticket type did the theater department sell? Calculate the solution to the system of linear equations using technology with matrices. Round decimals to four decimal places.