PAP Algebra 2

Unit 5

Square Root Functions, Inverse Functions, Exponent Rules

Name________________________

Period______
PAP Algebra II  
Unit 4B After Test Work  
Rational Exponents

\[ x^{\frac{1}{2}} = \sqrt{x} \quad x^{\frac{1}{3}} = \sqrt[3]{x} \quad x^{\frac{1}{4}} = \sqrt[4]{x} \quad x^{\frac{1}{n}} = \sqrt[n]{x} \]

\begin{align*}
a) \quad 25^{\frac{1}{2}} & = \quad \phantom{=} \\
b) \quad (64)^{\frac{1}{3}} & = \quad \phantom{=} \\
c) \quad (-64)^{\frac{1}{3}} & = \quad \phantom{=}
\end{align*}

\begin{align*}
d) \quad 16^{\frac{1}{4}} & = \quad \phantom{=} \\
e) \quad (-16)^{\frac{1}{4}} & = \quad \phantom{=} \\
f) \quad -16^{\frac{1}{2}} & = \quad \phantom{=}
\end{align*}

\[ x^{\frac{q}{n}} = \left( x^q \right)^{\frac{1}{n}} = \sqrt[n]{x^q} \]

\begin{align*}
a) \quad 8^{\frac{2}{3}} & = \quad \phantom{=} \\
b) \quad 9^{\frac{3}{2}} & = \quad \phantom{=} \\
c) \quad 625^{\frac{3}{4}} & = \quad \phantom{=} \\
d) \quad (-125)^{\frac{3}{3}} & = \quad \phantom{=}
\end{align*}

EX1: Write the following in simplified radical form:

\begin{align*}
a) \quad 4x^{\frac{2}{3}} & = \\
b) \quad (-3b)^{\frac{2}{5}} & = \\
c) \quad 4^{\frac{3}{2}} \cdot a^{-\frac{1}{2}} & = \\
d) \quad \left( \frac{-125m^3}{27n^4} \right)^{\frac{2}{5}} & = 
\end{align*}
EX2: Write the following with positive rational exponents:

a) \( \left( \sqrt[3]{-8z} \right)^2 = \)

b) \( \sqrt[5]{\frac{4z^3}{y^5}} = \)

EX3: Simplify the following expressions. Write all answers in radical form.

*When problems are written in fraction exponents, work the problem with the fraction exponents and then convert to radical form at the end after everything has been simplified.

a) \( \left( \frac{81a^2b^{-3}}{16a^{-2}b^{-1}} \right)^{\frac{3}{4}} \)

b) \( \frac{4n^{-\frac{2}{3}}(n^{\frac{8}{3}} - 3n^{\frac{5}{3}})}{2n^{\frac{1}{3}}} \)

c) \( \left( \frac{\sqrt[6]{4y}}{\sqrt[3]{4y}} \right)^2 \)

d) \( \sqrt[3]{\frac{-27q^{-1}r}{64q^2r^{-3}}} \)

*When the problems are written in radical form, convert to fraction exponents to work the problem and then back to radical form once simplified completely.
1) Fill in the following table of values using the function, \( f(x) = \sqrt{x} \), and graph the function on the following grid.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

2) Write the transformations of \( f(x) = -2\sqrt{x-4} - 3 \) in order.

3) Graph the following functions as transformations of \( y = \sqrt{x} \) and give the domain and range.

a) \( f(x) = 2\sqrt{x-3} \)

b) \( f(x) = -\sqrt{x} + 2 \)

c) \( f(x) = \sqrt{-x-3} \)

d) \( f(x) = 3\sqrt{x-5} + 2 \)
Graph #1-8 and find the domain and range.

1) \( y = \sqrt{x+3} - 1 \)

2) \( y = -\sqrt{x} + 3 \)

3) \( y = -3\sqrt{x-4} \)

4) \( y = \sqrt{-x} - 5 \)

5) \( y = -3\sqrt{x} - 4 \)

6) \( y = -\sqrt{x} + 5 \)

7) \( y = 2\sqrt{x-3} - 2 \)

8) \( y = -\sqrt{x} + 5 \)
9) Write the equation of the following graphs.

a) ![Graph A]

b) ![Graph B]

10) The function \( f(x) = \sqrt{x} \) is translated 2 units left and 5 units up. If \( g(x) \) represents the transformation of \( f(x) \), what is the equation of \( g(x) \)?

11) Under certain conditions, a skydiver's terminal velocity, \( v_t \) (in feet per sec) is given by

\[
v_t = 33.7 \frac{W}{A}
\]

where \( W \) is the weight of the skydiver and \( A \) is the skydiver's cross-sectional surface area (in sq. feet). Note that skydivers can vary their surface area by changing positions as they fall.

a) Write an equation for a skydiver who weighs 165 pounds.

b) Complete the table of values for the equation from part (a).

<table>
<thead>
<tr>
<th>( A )</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_t )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c) Use your table to graph the equation.
In order for a function to even have an inverse, it has to be one-to-one. This means that the graph must pass the horizontal line test, or the function can have only one x-value for each y-value.

Let's start with the function \( f(x) = x^3 + 2 \).

Is \( f(x) \) one-to-one? ______

If the function is indeed one-to-one, then it has an inverse. The way to find the equation for the inverse function is to switch the x and y in the equation and solve for y.

Start with this function: \( y = x^3 + 2 \)

Switch the x and y: \( x = y^3 + 2 \)

solve for y: \( x - 2 = y^3 \)

\( \sqrt[3]{x-2} = y \) \( \text{This is the inverse of } f(x) \text{ which is also called } f^{-1}(x) \). So \( f^{-1}(x) = \sqrt[3]{x-2} \).

**Properties of Inverses:**

1) If the point \((x, y)\) is on the graph of \( f(x) \), then the point \((y, x)\) is on the graph of \( f^{-1}(x) \).

2) The graph of \( f(x) \) and its inverse \( f^{-1}(x) \) are reflected across the line \( y = x \).

3) The domain for \( f(x) \) is the range for \( f^{-1}(x) \), and the range for \( f(x) \) is the domain for \( f^{-1}(x) \). (i.e. the domain and range are switched)

4) The composition of a function and its inverse will always equal x. \( f(f^{-1}(x)) = x \) or \( f^{-1}(f(x)) = x \)

Verify property #2 and #1 for the above example on the graph.
Verify all 4 properties for the following pairs of functions and their inverses.

EX1) \( f(x) = 3x - 6 \quad f^{-1}(x) = \frac{1}{3}x + 2 \)

PROPERTY 1:
PROPERTY 2:
PROPERTY 3:
PROPERTY 4:

If the function is one-to-one, find its inverse. Also find the domain and range for the function and it’s inverse. Restrict the domain when necessary.

1) \( f(x) = x^2 - 3 \)

DOMAIN: __________
RANGE: _________

One-to-one? _____

\( f^{-1}(x) = \) __________

DOMAIN: __________
RANGE: _________
2) \( f(x) = x^2 + 1 \quad x \geq 0 \)

DOMAIN: __________
RANGE: _________
One-to-one? _____

\[ f^{-1}(x) = \] __________

DOMAIN: __________
RANGE: _________

3) \( f(x) = \sqrt{x - 2} \)

DOMAIN: __________
RANGE: _________
One-to-one? _____

\[ f^{-1}(x) = \] __________

DOMAIN: __________
RANGE: _________
4) \( f(x) = \frac{1}{x - 3} \)

Domain: __________

Range: __________

One-to-one? _____

\[ f^{-1}(x) = \] __________

Domain: __________

Range: __________

5) Which of the following sets of relations are inverses? Why?

a) \( \{1,2,3,4,5\} \rightarrow \{9,8,7,6,5\} \) and \( \{9,8,7,6,5\} \rightarrow \{1,2,3,4,5\} \)

b) \[
\begin{array}{c|c}
  x & y \\
  \hline
  0 & -2 \\
  -3 & 9 \\
  7 & -11 \\
  -12 & -1 \\
  4 & 5 \\
\end{array}
\]

\[
\begin{array}{c|c}
  x & y \\
  \hline
  5 & 4 \\
  -2 & 0 \\
  -1 & -12 \\
  9 & -3 \\
  -11 & 7 \\
\end{array}
\]

c)

d) \( f(x) = \frac{1}{x - 2} \) and \( g(x) = \frac{1}{x + 2} \)
1) \( f(x) = 2x - 3 \)

a) Is the function \( f(x) \) one-to-one? ______

b) Find the domain and range for the function \( f(x) \).

\[
\text{Domain: } \quad \text{Range: } \quad
\]

c) If the function is one-to-one, find its inverse. Restrict the domain if necessary.

\[
f^{-1}(x) = \quad
\]

d) Graph the inverse.

e) Find the domain and range for the inverse function.

\[
\text{Domain: } \quad \text{Range: } \quad
\]

2) \( f(x) = x^2 - 4 \quad x > 0 \)

a) Is the function \( f(x) \) one-to-one? ______

b) Find the domain and range for the function \( f(x) \).

\[
\text{Domain: } \quad \text{Range: } \quad
\]

c) If the function is one-to-one, find its inverse. Restrict the domain if necessary.

\[
f^{-1}(x) = \quad
\]

d) Graph the inverse.

e) Find the domain and range for the inverse function.
3) \( f(x) = x^2 + 3 \quad x \leq 0 \)

a) Is the function \( f(x) \) one-to-one? 

b) Find the domain and range for the function \( f(x) \).

\[
\text{Domain: } \_\_\_\_\_\_\_\_ \quad \text{Range: } \_\_\_\_\_\_\_\_
\]

c) If the function is one-to-one, find its inverse. Restrict the domain if necessary.

\[
f^{-1}(x) = \_\_\_\_\_\_\_\_
\]

d) Graph the inverse.

e) Find the domain and range for the inverse function.

\[
\text{Domain: } \_\_\_\_\_\_\_\_ \quad \text{Range: } \_\_\_\_\_\_\_\_
\]

4) \( f(x) = \sqrt{x + 3} \)

a) Is the function \( f(x) \) one-to-one? 

b) Find the domain and range for the function \( f(x) \).

\[
\text{Domain: } \_\_\_\_\_\_\_\_ \quad \text{Range: } \_\_\_\_\_\_\_\_
\]

c) If the function is one-to-one, find its inverse. Restrict the domain if necessary.

\[
f^{-1}(x) = \_\_\_\_\_\_\_\_
\]

d) Graph the inverse.

e) Find the domain and range for the inverse function.

\[
\text{Domain: } \_\_\_\_\_\_\_\_ \quad \text{Range: } \_\_\_\_\_\_\_\_
\]
5) \( f(x) = \sqrt{-x} + 3 \)

a) Is the function \( f(x) \) one-to-one? ______

b) Find the domain and range for the function \( f(x) \).

Domain: ____________     Range: ____________

c) If the function is one-to-one, find its inverse. Restrict the domain if necessary.

\[ f^{-1}(x) = \] ____________

d) Graph the inverse.

e) Find the domain and range for the inverse function.

Domain: ____________ Range: ____________

6) \( f(x) = -\sqrt{x} + 4 \)

a) Is the function \( f(x) \) one-to-one? ______

b) Find the domain and range for the function \( f(x) \).

Domain: ____________ Range: ____________

c) If the function is one-to-one, find its inverse. Restrict the domain if necessary.

\[ f^{-1}(x) = \] ____________

d) Graph the inverse.

e) Find the domain and range for the inverse function.

Domain: ____________ Range: ____________
7) \( f(x) = \frac{1}{x} - 1 \)

a) Is the function \( f(x) \) one-to-one? _____

b) Find the domain and range for the function \( f(x) \).
   Domain: _______________     Range: _______________

c) If the function is one-to-one, find its inverse. Restrict the domain if necessary.
   \[ f^{-1}(x) = \quad \]

d) Graph the inverse.

e) Find the domain and range for the inverse function.
   Domain: _______________   Range: _______________

8) \( f(x) = \frac{1}{x + 1} \)

a) Is the function \( f(x) \) one-to-one? _____

b) Find the domain and range for the function \( f(x) \).
   Domain: _______________     Range: _______________

c) If the function is one-to-one, find its inverse. Restrict the domain if necessary.
   \[ f^{-1}(x) = \quad \]

d) Graph the inverse.

e) Find the domain and range for the inverse function.
   Domain: _______________   Range: _______________
Part 1
Axel is a Danish foreign exchange student. Since Celsius is the temperature scale in Denmark, Axel uses the function \( C(F) = \frac{5}{9} (F - 32) \) (or \( C = \frac{5}{9} (F - 32) \)) for converting from degrees Fahrenheit, \( F \), to degrees Celsius, \( C \). He derived the function from the freezing point (32, 0) and the boiling point (212, 100), where each ordered pair is (\( F, C \)) or (Fahrenheit, Celsius).

1. What are the independent and dependent variables in \( C = \frac{5}{9} (F - 32) \)? Explain.

   Independent:___________________ Dependent:___________________

2. Evaluate \( C(90) = \) _______ Write the ordered pair (\( F, C \)) and explain its meaning in the context of this problem.

   Now, you and Axel are taking a trip to Denmark, which uses the Celsius temperature scale. You need to write a function that will give an output of degrees Fahrenheit when using an input of degrees Celsius.

3. Now, what are the independent and dependent variables for this new function?

   Independent:___________________ Dependent:___________________

4. How are this function and the original function related?

5. Write the function for the new situation.
Part 2

1. Fill in the tables below for \( C = \frac{5}{9}(F - 32) \) and \( F = \frac{9}{5}C + 32 \).

\[
\begin{array}{|c|c|}
\hline
\text{Independent Variable (F)} & \text{Dependent Variable (C)} \\
\hline
23 & 5 \\
32 & 0 \\
86 & 30 \\
212 & 100 \\
\hline
\end{array}
\]

2. Describe the relationship between the tables for \( C = \frac{5}{9}(F - 32) \) and \( F = \frac{9}{5}C + 32 \).

3. If a function has the ordered pair \((a, b)\), the function’s inverse must contain what ordered pair?

In Part 1 of the activity we determined that the inverse of a function could be determined by switching the independent and dependent variables and isolating the dependent variable. When we write an equation in terms of \(x\) and \(y\), \(x\) is the independent variable and \(y\) is the dependent variable. Thus, the steps for finding an inverse of an equation written in terms of \(x\) and \(y\) are (a) switch the independent variable \(x\) and the dependent variable \(y\) (b) isolate \(y\) or solve for \(y\).

Find the inverse of the following functions.

4. \( y = 2x + 4 \)  
5. \( y = -3x - 10 \)  
6. \( y = \frac{2}{3}x - 4 \)
Solve each of the following equations or inequalities for the variable. Be sure to check for extraneous solutions.

1. $\sqrt{x} + 3 = 7$  
2. $\sqrt{x} + 2 + 1 = 4$  
3. $\sqrt{x} + 12 = 9$

4. $\sqrt{5x} + 1 = 4$  
5. $\sqrt{x} + 7 + \sqrt{x} = 7$  
6. $\sqrt[3]{6x + 10} = -8$

7. $\sqrt[3]{3x} + 1 = -2$  
8. $\sqrt{6x} - 2 \leq 10$  
9. $\sqrt{x} + 24 \leq 6$

10. $\sqrt{3x} - 6 < 2$  
11. $\sqrt{y} + 9 < \sqrt{3}$
12. \( d = \sqrt{1.5h} \) is the equation for the distance a submarine periscope can see (in miles) when it is \( h \) feet above the water. How far would a submarine periscope have to be above the water to see a ship 4 miles away?

13. After a bungee jumper jumps, they tend to swing back and forth for a few minutes. An equation for the time, \( t \), in seconds it takes a bungee jumper on a cord to make a complete swing is \( t = 2\pi \sqrt{\frac{l}{32}} \), where \( l \) is the length of the cord in feet.

   a) If the cord is 20 feet long how long does it take to complete one swing of the bungee jumper?

   b) How long would the cord need to be if you wanted to triple the time needed for the pendulum to complete one full swing?
Solve the following equations for the variable. Check all answers to eliminate extraneous solutions.

1) $\sqrt{5x+1} = 6$
2) $\sqrt{2x} - \frac{3}{2} = 0$
3) $-2\sqrt{24x} + 13 = -11$

4) $\sqrt{x-16} = 2$
5) $\sqrt{6x} + 1 < 10$
6) $-4\sqrt{x+10} + 3 = 15$

7) $\sqrt{x-25} + 3 \leq 5$
8) $\sqrt[3]{8x^3} - 1 = 2x - 1$
9) $\sqrt{4x+1} \leq \sqrt{x+10}$

10) $\sqrt{3x-8} + 1 = \sqrt{x+5}$
11) $\sqrt{2x+5} = \sqrt{x+2} + 1$
12) $\sqrt{2x+3} + 2 = \sqrt{6x+7}$
13) In order to compete in America’s Cup sailboat race, a boat must satisfy the rule \( l + 1.25\sqrt{s} - 9.8\sqrt{d} \leq 16 \) where \( l \) is the length in meters of the boat, \( s \) is the area in square meters of the sails and \( d \) is the volume in cubic meters of water displaced by the boat. If a boat has a sail length of 20 meters and displaces 27 cubic meters of water, what is the maximum allowable value for \( s \)?

14) The function \( t = \sqrt{5a} \) predicts the age of an artifact, \( a \), using the thickness of the layer of moisture \( t \) (in microns which are millionths of a meter). If the thickness of an artifact is 206.15 microns, how old is the artifact?

15) You collect the following data comparing the number of fox in different Texas state parks to the number of squirrels eaten in the parks.

<table>
<thead>
<tr>
<th>Fox</th>
<th>9</th>
<th>49</th>
<th>169</th>
<th>289</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squirrels eaten</td>
<td>6</td>
<td>14</td>
<td>26</td>
<td>34</td>
</tr>
</tbody>
</table>

- Which equation models the situation?
  a. \( S = \sqrt{2F} \)  
  b. \( S = \sqrt{3F} \)  
  c. \( S = 3\sqrt{F} \)  
  d. \( S = 2\sqrt{F} \)

- How many squirrels would be eaten if there were 529 fox?

- If 54 squirrels are eaten, how many fox are there in the park?

- What is the domain of this situation?

- What is the range?

16) The distance Jack Bauer can safely rescue Renee from Dubaku’s thugs (in meters) is related by a square root equation to the bandwith to which Chloe has access on the computer (in Kb/s). The data is listed below:

<table>
<thead>
<tr>
<th>Bandwith</th>
<th>0</th>
<th>10</th>
<th>40</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

- Which equation models the situation?
  a. \( D = \sqrt{2B} \)  
  b. \( D = \sqrt{3B} \)  
  c. \( D = \sqrt{2.5B} \)  
  d. \( D = \sqrt{3.5B} \)

- How far away can Jack be and still rescue Renee if Chloe has access to 160 Kb/s of bandwith?

- If Chloe has 250 Kb/s of bandwith, how far away can Jack be and still rescue Renee?

- What is the domain of this situation?

- What is the range?